

Name \_\_\_\_\_ ID \_\_\_\_\_

**Activity 3-1 (15 Jul 2021)**

1. Prove the following statement:

If integer  $c$  divides both integers  $a$  and  $b$ , then  $c$  divides  $a - b$ .

*(Hint: try direct proofs.)*

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**Activity 3-2 (15 Jul 2021)**

2. Prove the following statement: If  $x$  is irrational, then  $\sqrt{x}$  is irrational.  
(Hint: try proofs by contraposition.)

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**Activity 3-3 (15 Jul 2021)**

3. Assume that  $x$  is a non-zero rational number. Prove that if  $y$  is irrational, then  $xy$  is irrational.  
(Hint: try proofs by contraposition.)

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**Activity 3-4 (15 Jul 2021)**

4. Prove that for any positive integer  $n$ ,  $n$  is an odd number if and only if  $5n + 6$  is odd.

(Hint: To prove statement  $P \Leftrightarrow Q$ , you can prove that  $P \Rightarrow Q$  and  $Q \Rightarrow P$ .)

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**Activity 3-5 (15 Jul 2021)**

5. Prove the following statement: If  $x$  and  $y$  are integers and  $x^2 + y^2$  is even, then  $x + y$  is even.

*Note: When you want to prove this statement: "If  $x$  and  $y$  are integers and  $x^2 + y^2$  are even, then  $x + y$  is even. ". You can think of it as: "If  $x$  and  $y$  are integers, then (if  $x^2 + y^2$ , then  $x+y$  is even)". That is because  $(P \text{ and } Q) \Rightarrow R$  is equivalent to  $(P \Rightarrow (Q \Rightarrow R))$ . Therefore, in this case, you can start by assuming that  $x$  and  $y$  are integers.*

*(Hint: proofs by cases.)*