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# Activity 9

9.1 Consider vectors over field $\mathbb{R}$. Let $\boldsymbol{u}_1 = [1, 1, 0, 0]$ and $\boldsymbol{u}_2 = [0, 1, 1, 0]$. Show that the following vectors are in Span $\{\boldsymbol{u}_1, \boldsymbol{u}_2\}$.

(a) $[1, 1, 0, 0]$

(b) $[1, 2, 1, 0]$

(c) $[1, 0, -1, 0]$

9.2 Consider vectors over field $GF(2)$. Let $\boldsymbol{u}_1 = [1, 1, 0, 0]$, $\boldsymbol{u}_2 = [0, 1, 1, 0]$, and $\boldsymbol{u}_3 = [1, 0, 1, 0]$.

- Find Span $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$.
- Let $\mathcal{V} = \text{Span}\ \{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$. Find a vector in $GF(2)^4$ which is not in $\mathcal{V}$.
- What is $\dim \mathcal{V}$?

9.3 Consider vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_{k-1}$ over $\mathbb{R}$. Let $\boldsymbol{v}_k = \boldsymbol{v}_1 + \boldsymbol{v}_2 + \ldots + \boldsymbol{v}_{k-1}$. Show, from the definition of linear independence, that the set of vectors $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k\}$ are not linearly independent.

9.4 Consider vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k$ over $\mathbb{R}$. Let $\mathcal{V} = \text{Span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k\}$.

Suppose that there exists a non-zero vector $\boldsymbol{u} \in \mathcal{V}$ that can be written as linear combinations of $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k$ in two ways, i.e., there exists $\alpha_1, \alpha_2, \ldots, \alpha_k \in \mathbb{R}$ and $\beta_1, \beta_2, \ldots, \beta_k \in \mathbb{R}$ such that

$$\boldsymbol{u} = \alpha_1 \boldsymbol{v}_1 + \alpha_2 \boldsymbol{v}_2 + \cdots + \alpha_k \boldsymbol{v}_k,$$

$$\boldsymbol{u} = \beta_1 \boldsymbol{v}_1 + \beta_2 \boldsymbol{v}_2 + \cdots + \beta_k \boldsymbol{v}_k,$$

and also $\alpha_i \neq \beta_i$, for some $1 \leq i \leq k$.

Prove that $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k\}$ is **not** a basis of $\mathcal{V}$.

*(Notes: In this problem, you must prove this fact from standard definitions. You cannot just use a theorem we proved in class to claim the proof.)*