

Name \_\_\_\_\_ ID \_\_\_\_\_

**Activity 3-1 (23 Aug 2018)**

1. [Proof by cases] Prove the following statement: If  $x$  and  $y$  are integer and  $x^2 + y^2$  is even, then  $x + y$  is even.

Note: When you want to prove this statement: "If  $x$  and  $y$  are integers and  $x^2 + y^2$  are even, then  $x + y$  is even. ". You can think of it as: "If  $x$  and  $y$  are integers, then (if  $x^2 + y^2$ , then  $x+y$  is even)". That is because  $(P \text{ and } Q) \Rightarrow R$  is equivalent to  $(P \Rightarrow (Q \Rightarrow R))$ .

Therefore, in this case, you can start by assuming that  $x$  and  $y$  are integers.

2. (source: LPV) Prove by induction on  $k$  that for any integer  $k \geq 1$ , we have that

$$1+3+\dots+(2k+1)=k^2$$

State the property  $P(k)$ :

$P(k)$	
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**Basic step:** (show that  $P(1)$  is true)

**Induction step:** (assume  $P(m)$  and show  $P(m+1)$ , for any  $m \geq 1$ )

State the Induction Hypothesis  $P(m)$ :

$P(m)$	
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State the goal  $P(m+1)$

$P(m+1)$	
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