

01204213: Homework 3

Due: 23pm, 26 Jul 2021.

1. (Siper 1.20) For each of the following languages, give two strings that are members and two strings that are not members—a total of four strings for each part. Assume the alphabet $\Sigma = \{\mathbf{a}, \mathbf{b}\}$ in all parts.

- (a) $\mathbf{a^*b^*}$
- (b) $\mathbf{a(ba)^*b}$
- (c) $\Sigma^* \mathbf{a} \Sigma^* \mathbf{b} \Sigma^* \mathbf{a} \Sigma^*$
- (d) $(\varepsilon \cup \mathbf{a})\mathbf{b}$
- (e) $(\mathbf{a} \cup \mathbf{ba} \cup \mathbf{bb})\Sigma^*$

2. (Sipsers 1.28) Convert the following regular expressions to NFAs using the procedure given in class. In all parts $\Sigma = \{\mathbf{a}, \mathbf{b}\}$.

- (a) $\mathbf{a(abb)^* \cup b}$
- (b) $\mathbf{a^* \cup (ab)^*}$
- (c) $(\mathbf{a} \cup \mathbf{b^*})\mathbf{a^*b^*}$

3. Let $F = \{ww \mid w \in \{0, 1\}^*\}$. Prove that F is not regular. (*Hint: choose the appropriate $s \in F$.*)

4. (Sipsers 1.51) Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection, and complement.

- (a) $\{0^n 1^m 0^n \mid m, n \geq 0\}$
- (b) $\{w \mid w \in \{0, 1\}^* \text{ is not a palindrome}\}$
- (c) $\{wtw \mid w, t \in \{0, 1\}^+\}$

Notes: A *palindrome* is a string that reads the same forward and backward. For example, 00100, 1, and 11 are palindromes, but 01 and 10011 are not.

Hints: For 3(b), you can use the fact that the complement of a regular language is regular.

5. (Sipsers 1.58) Let N be an NFA with k states that recognizes some language A . Show that if A is nonempty, A contains some string of length at most k .