Name_____**ID** _____

Activity 1-2 (9 Aug 2018)

1. Predicates (from wikibook
https://en.wikibooks.org/wiki/Discrete_Mathematics/Logic/Exercises#Logic_Exercise_5)
The following predicates are defined:

- friend(x) is "x is a friend of mine"
- wealthy(x) is "x is wealthy"
- clever(x) is "x is clever"
- boring(x) is "x is boring"

With these predicates, you can write "John is clever" as clever(John).

(a) Write each of the following propositions using predicate notation:

1.1 Jimmy is a friend of mine.	
1.2 Sue is wealthy and clever.	
1.3 Jane is wealthy but not clever.	
1.4 Both Mark and Elaine are friends of mine.	
1.5 If Peter is a friend of mine, then he is not boring.	
1.6 If Jimmy is wealthy and not boring, then he is a friend of mine.	

(b) Using the same predicates defined in question 1, symbolize each of the following.

(a) Some of my friends are clever.	
(b) All clever people are boring.	
(c) None of my friends is wealthy.	
(d) Some of my wealthy friends are clever.	
(e) All my clever friends are boring.	
(f) All clever people are either boring or wealthy.	

Quantifiers

2. Consider the universe to be "everything." For each of these statements, define appropriate predicates can rewrite the statement using the defined predicates and quantifiers. (Some predicate may have more than one variables)

Statements (and your answers)	Define your predicates here.
1 Every human must die.	
2 Some animal eats other animals.	
3 If a student works hard, that student will be successful.	
For questions 4 and 5, consider the universe to be a set of all people. 4 Everyone has someone that care about him or her.	
5 There is someone that everyone cares about.	
For questions 6 and 7, consider the universe to be a set of all companies. 6 When the economy is good, any companies can make good profits.	
7 When the economy is bad, only companies that can adapt survive.	

3. For each quantified proposition you answer in question 2, find its negation and translate the negation back to English.

Notes: *You may need to use a few logical equivalences to complete this questions, for example,* $P \Rightarrow Q \equiv \neg P \vee Q$.

	Negations	in English
1		
2		
3		

	Negations	in English
4		
5		
6		
7		

4. It seems that universal quantifiers are stronger than existential ones. Is it true that for any set A and predicate $P(x)$, i.e., is it true that

$$(\forall x \in A, P(x)) \Rightarrow (\exists x \in A, P(x))$$

Explain your reasoning and if it is not true, give a counter example.

5. (a) Use a truth table to show that $P \Rightarrow Q$ is equivalent to $\neg P \vee Q$.

(b) Use this equivalence to prove that $P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$