Name ID
Activity 2-2 (16 Aug 2018)

## Inference rules

4. Use inference rules and standard logical equivalences to show that hypotheses

$$
\begin{aligned}
& P \Rightarrow Q \\
& P \Rightarrow \neg Q
\end{aligned}
$$

leads to the conclusion $\neg P$.

| Steps | $\underline{\text { Reason }}$ |
| :--- | :--- |
|  |  |
|  |  |

5. Using inference rules to argue that if we assume

$$
\begin{aligned}
& \neg P \Rightarrow Q \\
& (P \vee R) \Rightarrow \neg S \\
& W \Rightarrow S \quad \text {, and } \\
& \neg Q
\end{aligned}
$$

then we can conclude that $W$ is false.

| Steps | Reason |
| :--- | :--- |
|  |  |
|  |  |

## Proofs (hints: try using direct proofs and proofs by contrapositions)

6 . Prove the following statement:
If integer $c$ divides both integers $a$ and $b$, then $c$ divides $a-b$.
7. Prove the following statement: If $x$ is irrational, then $\sqrt{x}$ is irrational.
8. Assume that $x$ is a non-zero rational number. Prove that if $y$ is irrational, then $x y$ is irrational.
9. Prove that for any positive integer $n, n$ is an odd number if and only if $5 n+6$ is odd.
(Hint: To prove statement $P<>Q$, you can prove that $P \Rightarrow Q$ and $Q=>P$.)
Write your proofs below

