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Activity 3-2 (23 Aug 2018)

3. In this problem, we will try to reconstruct Euclid's proof that there are infinitely many primes. We will prove by contradiction. So let's get you started.

The first step is to assume that there are finitely many primes. Let *n* be the number of prime numbers, and $p_1, p_2, \dots, p_n$ are all prime numbers.

The key to obtain the contradiction is to consider this number *L* defined to be

$$L = p_1 \times p_2 \times \cdots \times p_n + 1$$

Use this starting point to complete the proof. (*Hint:* What can you say about *L*? Is it a prime number? Do we really need to know for sure if it is a prime number?)

4 (MN) Prove that for any integer $n \geq 0$, the following formula is true:

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1 \quad .$$

State the property *P*(*n*):

P(*n*)	

5. Prove this statement. (Note that you do not have to use proof by contradiction.)

Suppose that I have k pairs of socks (each pair with a distinct color). If I pick $k + 1$ socks, then there will be at least one pair of socks with the same color.

6. (MN-ex-1b) Prove that for integer $n \geq 1$,

$$\sum_{i=1}^{n} i \cdot 2^{i} = (n-1)2^{n+1} + 2 \quad .$$

Name____**ID** __________________________

7. (R-3.3-ex-12) Prove that $3^n < n!$ whenever n is a positive integer greater than 6.

8. (LPV-2.1.5) Prove the following identity:

$$1\cdot 2+2\cdot 3+3\cdot 4+\cdots+(n-1)\cdot n=\frac{(n-1)\cdot n\cdot (n+1)}{3}$$

9. (LPV-2.5.4b) Prove that for any integer $n \geq 1$, $n^3 - n$ is a multiple of 6.