01204211: Exercises 8-2

Notes: To avoid confusion, in your answer, you can write vectors with this notation $\vec{u}$.

5. *Span test 3.* Consider 3-vectors over $\mathbb{R}$. Let $\boldsymbol{u}_1 = [1,2,3], \boldsymbol{u}_2 = [1,1,1], \boldsymbol{u}_3 = [1,2,2]$. We would like to show that $\boldsymbol{v} = [10,13,29] \in \text{Span}\ \{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$. However, in this case, it does not look very easy. Let us define variables $\alpha_1, \alpha_2, \alpha_3$ such that $\boldsymbol{v} = \alpha_1\boldsymbol{u}_1 + \alpha_2\boldsymbol{u}_2 + \alpha_3\boldsymbol{u}_3$. Write a system of linear equations for solving for these α_i's.

6. *Gaussian Elimination.* You can solve for $\alpha_1, \alpha_2, \alpha_3$ in the previous questions using Gaussian Elimination, that you learn from high school. Look up on the Internet to refresh your memory.

   Look up the on-line solver and use it to find the required α_i's. Write the solution down here (along with the reference to the website that you use):

7. *Unsolvable cases.* As in the previous questions, consider vectors in $\mathbb{R}^3$. In this question, your task is, however, to come up with examples where you cannot solve the span test problem.

   Define $\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3 \in \mathbb{R}^3$, and $\boldsymbol{v} \in \mathbb{R}^3$ such that $\boldsymbol{v} \notin \text{Span}\ \{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$. Explain briefly why it is the case.

8. *Infinite solution cases.* As in the previous questions, consider vectors in $\mathbb{R}^3$. In this question, your task is, however, to come up with examples where there are many combinations of α_i's, i.e., the representation is not unique.

   Define $\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3 \in \mathbb{R}^3$, and $\boldsymbol{v} \in \mathbb{R}^3$ such that $\boldsymbol{v} \in$ Span $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$ and there are many ways to write $\boldsymbol{v}$ as a linear combination of $\boldsymbol{u}_1, \boldsymbol{u}_1, \boldsymbol{u}_3$.

   Explain briefly why it is the case.