Name____**ID** ________________________

Activity 2-2 (16 Aug 2018)

<u>Inference rules</u>

4. Use inference rules and standard logical equivalences to show that hypotheses

$P \Rightarrow Q$

$P \Rightarrow \neg Q$

leads to the conclusion $\neg P$.

Steps	Reason

5. Using inference rules to argue that if we assume

$\neg P \Rightarrow Q$

$(P \vee R) \Rightarrow \neg S$

$W \Rightarrow S$, and

$\neg Q$

then we can conclude that W is false.

Steps	Reason

Proofs (hints: try using direct proofs and proofs by contrapositions)

6. Prove the following statement:

If integer c divides both integers a and b, then c divides $a - b$.

7. Prove the following statement: If x is irrational, then $\sqrt{x}$ is irrational.

8. Prove the following statement: If x and y are integer and $x^2 + y^2$ is even, then $x + y$ is even.

Note: When you want to prove this statement: "If x and y are integers and $x^2 + y^2$ are even, then x + y is even. ". You can think of it as: "If x and y are integers, then (if $x^2 + y^2$, then x+y is even)". That is because (P and Q) => R is equivalent to (P => (Q => R)). Therefore, in this case, you can start by assuming that x and y are integers.

9. Assume that x is a non-zero rational number. Prove that if y is irrational, then xy is irrational.

10. Prove that for any positive integer n, n is an odd number if and only if $5n + 6$ is odd.
(Hint: To prove statement P <=> Q, you can prove that P => Q and Q => P.)

Write your proofs below